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MAINTENANCE POLICY FOR OIL-LUBRICATED SYSTEMS WITH OIL ANALYSIS DATA

POLITYKA UTRZYMANIA RUCHU UKŁADÓW SMAROWANYCH OLEJEM W OPARCIU O DANE Z ANALIZY OLEJU

Maintenance of oil-lubricated systems plays a significant role in reducing unexpected system failures and improving machine availability. This paper deals with the oil-lubricated systems subject to gradual degradation that revealed by metal wear debris monitored using oil analysis. Oil-lubricated systems usually undertake several preventive maintenances during operation, after each maintenance, the system typical restores to an intermediate state between good-as-new state and bad-as-old state due to system aging such as cumulative wear. Furthermore, oil-lubricated systems often operate continuously in mission execution with availability constraints. However, existing literature still lacks a method to integrate the availability constraints with the system aging into the cause of optimizing the maintenance policy. To fill this gap, this paper develops a maintenance policy optimization method to determine the optimal maintenance threshold joint considering the availability constraints and the system aging. A case study of the power-shift steering transmission systems modelled by a wiener process is presented to illustrate the proposed method in practical application.

Keywords: oil-lubricated system; maintenance policy; oil analysis; availability constraint; system aging.

Konserwacja układów smarowanych olejem odgrywa istotną rolę w eksploatacji maszyn, umożliwiając zmniejszenie liczby nieoczekiwanych uszkodzeń i poprawiając dostępność maszyn. Niniejszy artykuł dotyczy układów smarowanych olejem ulegających stopniowej degradacji, którą można mierzyć za pomocą analizy oleju, monitorując zawartość drobin metalu powstających na skutek zużycia. Podczas swojej pracy, układy smarowane olejem zwykle poddawane są kilkakrotnie przeglądom zapobiegawczym. Po każdej konserwacji, układ wraca do stanu pośredniego między stanem fabrycznej nowości (as good as new) a stanem "jak przed konserwacją" (as bad as old), co wynika ze starzenia się systemu, m.in. skumulowanego zużycia. Co więcej, systemy smarowane olejem często działają w sposób ciągły, wykonując misje z ograniczeniami dostępności. Jednak w istniejącej literaturze wciąż brakuje metody, która pozwalałaby na zintegrowane ujęcie ograniczeń dostępności i starzenia w celu optymalizacji polityki utrzymania ruchu. Aby wypełnić tę lukę, w niniejszym artykule opracowano metodę optymalizacji polityki utrzymania ruchu, dzięki której można określić optymalny próg konserwacji z uwzględnieniem zarówno ograniczeń dostępności jak i starzenia się systemu. Możliwość praktycznego zastosowania zaproponowanej metody zilustrowano na podstawie studium przypadku układów przekładni kierowniczych zamodelowanych za pomocą procesu Wienera.

Słowa kluczowe: układ smarowany olejem; polityka utrzymania ruchu; analiza oleju; ograniczenie dostępności; starzenie się systemu.

Acronyms

CBM: condition-based maintenance LCM: lubricant condition monitoring MLE: maximum likelihood estimation PCA: principal component analysis PHM: prognostics and health management PM: preventive maintenance PSST: power-shift steering transmission

1. Introduction

Lubricant condition monitoring (LCM) has been widely used in oil-lubricated machinery benefit from the advance in analytical instruments and monitoring techniques, see, e.g., [2, 8, 14, 16, 22, 24] and references therein. The analysis results of oil samples not only give an indicator of the suitability of the lubricating oil for continued use but also provide information about the wear condition of oil-lubricated machines [20, 31]. This advantage enables the oil analysis technique to become one of the most popular data acquisition techniques used in condition monitoring, prognostics, and maintenance. For a comprehensive review of the use of oil analysis data for maintenance decision support, readers are referred to [28] and the references therein. In these existing studies, many efforts have been made in establishing a reasonable degradation model or estimating an accurate residual life. However, the existing literature still lacks an effective method to utilize oil analysis data to implement optimal maintenance to ensure the operation safety, availability and economic gains of the oil-lubricated systems. Therefore, this paper aims to fill this literature gap by addressing the condition-based maintenance (CBM) problem for the oillubricated systems with selected oil analysis data.

Spectral oil analysis is one of the most widely used oil analysis techniques to observe the increasing trends of wear debris concentrations in the oil-lubricated systems [9]. It can be used to identify the severity of wear conditions and even the impending system failure without dismantling the machine [24,30]. Specifically, if the oil spectral data reaches a predetermined failure threshold, the oil-lubricated system is considered as failed that should be maintained or repaired [19,31]. As shown in Fig. 1, for example (Liu et al., [15]), the oil spectral data, Cu, is used to monitoring the degradation of a Power-Shift Steering Transmission (PSST) system; the PSST system experiences degradation during operation and eventually fails (unable to operation satisfactorily) and needs to be replaced when the spectral oil data, X(t), reaches the threshold, D_F , at time t_F . If the degradation profile of the oil-lubricated system can be modelled and evaluated, the preventive maintenance (PM) action might be implemented based on the collected oil spectral data before the system failed.



Fig. 1. Condition monitoring using spectral oil data

Accordingly, to achieve an optimal implementation of the PM actions on oil-lubricated systems, there are two essential requirements must be addressed: (1) a reliable degradation model that can characterize the degradation profile of oil-lubricated systems, such that the degradation condition can be accurately estimated and provided as an input for PM; and (2) a reasonable maintenance objective function that can optimize the PM actions of oil-lubricated systems, such that the operational requirements of the maintained system can be better satisfied.

Over the past few years, many research efforts have been made on the requirement (1). Many types of degradation models, such as proportional hazard models [6,36], hidden Markov models [5,29], and Wiener process models [26,31], have been developed in modelling the degradation evolution of the oil-lubricated systems. Among these models, the Wiener process model can provide a good description of the system degradation process by describing the degradation as a non-monotonic process. Vališ et al. [27] considered using the wiener process with the selected spectral oil data (i.e., Fe, and Pb) to predict the expected moment when a soft system failure occurred and proved that the spectral oil data could provide a useful reference for CBM. In this paper, it is assumed that the degradation process follows a normal distribution by regarding the degradation as an increment of a large amount of small wear. Yan et al. [31] proposed a Wiener process-based degradation model for assessing the material wear of the friction pairs in an oil-lubricated system, and in [34] further investigated the influences of the spectral measurement uncertainty in modelling the system degradation. It is known that the collected oil spectral data usually shows a non-monotonic trend because of measurement uncertainty. Therefore, in this paper, the degradation profile of oil-lubricated systems is described using a Wiener process.

With the degradation profile predicted, PM (e.g., dismantling inspection, lubricating oil replacement) will be carried out. While extensive work has been done in developing these degradation models, the current literature still falls short in addressing the requirement (2), i.e., how to construct a reasonable maintenance objective function for the oil-lubricated systems. Most of the existing studies [32, 33, 35] simply use a cost minimization function as the objective, and the maintenance costs (e.g., oil analysis cost, oil replacement cost, dismantling inspection cost and system failure cost) are minimized in a full life cycle. However, for some critical systems in military equipment, mining machinery, and power industry, whose failure is critically hazardous and often leads to catastrophic consequences, and therefore, using a cost minimization objective in these cases may be problematic [1, 12]. In these critical systems, the ratio of time on the operation (uptime) compared with the time on the maintenance (downtime) is more important than the cost [3, 7]. Thus, in this paper, a more practical objective function, the maximum availability, is utilized to measure maintenance effectiveness.

Since the degradation of the oil-lubricated system is described by a wiener-process-based model, and the maintenance objective is constructed as an availability function, the PM threshold is set as the decision variable forming the optimal maintenance policy. In order to optimize the policy, the effect of each PM action should also be addressed. In the literature of CBM, the effect is usually divided into perfect PM and imperfect PM. However, the existing research concerning the oil-lubricated systems [32, 33, 35] simply assumes that the PM action is perfect without considering the system aging, and the maintained system will be recovered to a good-as-new state. Actually, using a perfect PM assumption for the oil-lubricated systems is problematic, as discussed in many related studies [4,10]. For instance, as discussed in Yan et al. [31], "because of the system residual damage such as cumulative wear, the degradation will start at some nonzero value after each PM and randomly increasing with the order of PM cycle." This phenomenon is called the system aging property that will shorten the time interval to the next PM. To address this issue, in this paper, the PM action is assumed imperfect and can only partially restore the system, i.e., recover the system to an intermediate state between bad-as-old state and good-as-new state.

Motivated by the above observations, in this paper, an optimal PM problem is considered in terms of the availability requirement considering the system aging property. The main objective of this paper is to develop a maintenance decision method for oil-lubricated systems based on oil analysis data in order that the PM threshold can be achieved with maximum availability. Compared with the existing studies in Liu et al. [35] and Yan et al. [32], the proposed method has the following innovations: (1) considering the system availability requirement as the objective to PM policy optimization; and (2) involving the system aging property in the optimization model. To be specific, the oil-lubricated systems that monitored under oil spectral analysis, with an availability requirement subject to periodic unperfect PM (e.g., lubricating oil replacement), are considered. For modelling the PM process, a wiener process is utilized for the degradation model, and a geometric process is used for the system residual damage after imperfect PM actions. In addition, the availability requirement under the short-run availability constraint is adopted in the optimal PM policy model. Finally, a case study for several PSST systems in military vehicles is provided to illustrate the proposed method. The results show the practicality, effectiveness and robustness of the proposed method.

The framework of the proposed maintenance policy optimization method is shown in Fig. 2. The remainder of the paper is structured as follows. Section 2 gives the motivation of the concerned optimal PM problem for oil-lubricated systems and its properties under multiple imperfect PM action. The system aging property description, the operation time model, and the maintenance duration model are introduced in Section 3. Section 4 gives the formulation of the optimization problem for the oil-lubricated system and the corresponding calculation procedures for achieving the optimal PM policy. In Section 5, a case study for several PSST systems is provided for illustration of the proposed method. Finally, conclusions and discussions are drawn in Section 6.



Fig. 2. Framework for the maintenance policy optimization

2. PM policy for oil-lubricated systems

This section provides a general description of the concerned optimal PM problem for oil-lubricated systems. This paper deals with the oil-lubricated systems that degrade over time in mission execution conditions, and oil spectral analysis is periodically conducted to monitor the degradation severity of the system during the whole lifecycle (from an initial to failure). As shown in Fig. 3, the degradation process $\{X(t), t \ge 0\}$ that measured using collected spectral oil data is used to predict the degradation profile of the oil-lubricated system. When the degradation profile of the oil-lubricated system reaches a predetermined PM threshold, D_{PM} , at the end of a mission, PM action will be implemented immediately before the next mission (e.g., at time $R_{i=1,2,3}$ in Fig. 3), and the time for the PM actions is M_i . Alternatively, the oil-lubricated system will continue to operate with no PM action until the next oil inspection. Please note that the PM actions mainly include the lubricating oil replacement, the possible dismantling inspection, and the replacement of the potential components, usually are imperfect, i.e., the PM actions cannot recover the oil-lubricated system to the good-as-new state. When a new one replaces the used oil-lubricated system (e.g., at time t_0 in Fig. 3), the system will be fully restored to a good-as-new state and a time ζ æ is incurred.



Fig. 3. State transition of an oil-lubricated system with PM and system replacement

Let T_i (i = 1, 2, ...) represents the operating time of the oil-lubricated system after a PM action. In reality, the expected up-

time period, $E[T_i]$, for the maintained oil-lubricated system to reach the PM threshold in a cycle show a decreasing trend because of the imperfect nature of PM and the system aging such as cumulative wear. On the other hand, the expected downtime period, $E[M_i]$, for the maintained oil-lubricated system to perform each PM action in a cycle usually show an increasing trend, since a more extended period is required to maintain a severely degraded system than a healthier one. However, for the oil-lubricated systems used in military equipment, mining machines, and power industries, the mission interruption time is strictly limited. As a result, the purpose of this research is to find the optimal PM policy to maximize the mission availability of an oil-lubricated system, that is, to maximize the expected uptime compared with the expected downtime.

Based on the above description of the concerned optimal PM problem, the mission availability of the oil-lubricated system shows a decreasing trend in a replacement cycle, since the expected downtime increases whereas the expected uptime decreases. Therefore, the mission availability metric in Eq. (1) is adopted as the optimization objective to assess the effectiveness of the maintenance policy. The mission availability for oil-lubricated systems in a life cycle is defined as:

$$Availavility_L = \frac{(\text{expected uptime})/\text{cycle}}{(\text{expected uptime} + \text{expected downtime})/\text{cycle}} (1)$$

where the "cycle" is defined as the period between two consecutive replacements of the oil-lubricated systems, the "downtime" includes the time for all PM actions and system replacement. Besides, for oil-lubricated systems, frequent PM actions are not allowed in engineering practice. Thus, the oil-lubricated systems must satisfy a minimum short-run availability. Specifically, the average short-run availability metric defined in (2) is adopted to represent the mission constraint of an oil-lubricated system after the *i* th PM action, within a replacement life cycle, as described above.

Availavility
$$S(i) = \frac{expected uptime after ith PM}{(expected uptime + expected downtime)after ith PM}$$
(2)

As many researchers have investigated, a system will no longer be suitable for operation when the defined average short-run availability decreases to a certain level [4,10,31]. Thus, whenever the short-run availability, *Availavility* $_S_{min}$, is lower than a threshold after the *N* th PM action, a system replacement will be conducted (e.g., at time R_4 in Fig. 3) and, the oil-lubricated system would be restored to a good-as-new state. Specifically, the replacement policy for the oillubricated system is summarized as:

$$\begin{cases} Availavility_S(i) > Availavility_S_{min} & i \in [1, N-1] \rightarrow \text{PM actions} \\ Availavility_S(N) \le Availavility_S_{min} & \rightarrow \text{System replacement} \end{cases}$$
(3)

Remark 1: The objective of this paper is to determine the optimal value of the PM threshold, D_{PM} , to maximize the achieved availability defined in Eq. (1) for a required service time under the constraint of the average short-run availability defined in Eq. (2). We refer to this newly proposed policy as availability limit policy. Unlike the existing cost limit policy [25-27], where the maintenance cost is assumed as the optimization objective, the proposed policy considering the mission availability to initiate the PM actions and system replacement. In this way, the achieved availability of the maintained oil-lubricated system in the whole mission execution can be maximized, which is a practical advantage for the maintenance of oil-lubricated systems used in critical systems, such as the PSST system in armored tank vehicles [31], the wind turbine gearbox portion of wind power systems [23], and the diesel engine in marine freighters [11].

Equation (3) can be numerically calculated using a searching algorithm. From the above description, it is conculcated that the optimal maintenance policy is heavily dependent on the system aging property, and the expected uptime and downtime, which can be fitted by the historical time data of system operation. Thus, in the following, the residual damage, and operation time model and maintenance time model are investigated. The aim is to accurately initiate the PM policy optimization model and obtain the optimal PM threshold.

3. Imperfect PM model considering system aging

In order to establish the considered maintenance optimization model, it is necessary to investigate the influence of the aging property of oil-lubricated systems. According to the system aging characteristics described in Section 2, the system aging property, the required operation time model and PM duration model involving multiple PM actions are given as follows.

3.1. The System Aging Property

Recall that the PM actions would not restore the maintained oillubricated system to a good-as-new state but an intermediate state due to the system aging, i.e., cumulative wear of friction pairs. Thus, the characteristics of the aging propriety after a single PM action and the relationship with multiple PM actions are investigated for the maintained oil-lubricated system.

In general, the PM actions will at least restore the maintained system to a healthier condition than the state before the PM. Fig. 4 provides a schematic diagram of a viable imperfect PM model to describe the PM effect considering the system aging. That is, the residual damage, $X(R_i^+)$, of the maintained system after each PM action in a



Fig. 4. System aging property with imperfect PM

It is commonly assumed that the residual damage in a replacement cycle follows a stochastic process with an exponentially increasing mean and a constant variance. Actually, such an assumption is widely used in modeling the aging property of the degrading systems, as shown in many applications [3, 4, 10]. To be specific, the mean and variance are given as follows:

$$E\left[\frac{X(R_i^+)}{D_{PM}}\right] = 1 - \exp(-iu) \tag{4}$$

$$\left| Var \left[\frac{X \left(R_i^+ \right)}{D_{PM}} \right] = \sigma^2$$
⁽⁵⁾

where $u \ge 0$ is a constant value describing the effort of the PM actions, $\sigma^2 \ge 0$ is a constant value describing the variance of the PM actions.

Remark 2: The system aging property is represented by the proposed residual damage model, which can be understood in the following way. If [u > 0, $\sigma^2 > 0$] is assumed, the residual damage, $X(R_i^+)$, becomes a random variable constrained in a finite interval [$0, D_{PM}$]. In this case, the probabilistic models with compact support including the *Beta* distribution are commonly used to describe the system aging property [13]. When [u > 0, $\sigma^2 > 0$] is satisfied, the maintenance effect results in restoring the system health condition depending on the selection of D_{PM} . On the other hand, [u = 0, $\sigma^2 = 0$] refers to the perfect maintenance that recovers the maintained system back to the good-as-new state. Obviously, such a perfect PM assumption is a special case of the proposed system residual damage model, as have used in the existing literature [33, 32].

3.2. The operation time model

Since the system aging property of the maintained oil-lubricated system is determined by using the residual damage model, the expectation of the operating time after the (i + 1) th PM action, T_{i+1} , can be calculated with a given D_{PM} , namely:

$$E[T_{i+1}] = E\left[E\left[T_{i+1} \mid X\left(R_{i}^{+}\right)\right]\right] = \int_{0}^{D_{PM}} E\left[T_{i+1} \mid X\left(R_{i}^{+}\right)\right] f_{X\left(R_{i}^{+}\right)}(X) dx \quad (6)$$

where $f_{X(R_i^+)}$ is the probability density function (PDF) of the residual damage, $X(R_i^+)$. Recall that probabilistic models with compact support are commonly used to describe the residual damage. Thus, in this paper, the *Beta* distribution is employed to depict the residual damage, $X(R_i^+)$, just like many researchers have been done [3, 4, 7, 10, 13]. Such that the PDF series, $f_{X(R_i^+)}(X)$, is then defined as:

$$f_{X\left(R_{i}^{+}\right)}\left(X\right) = \frac{1}{D_{PM}} \frac{\Gamma\left(\alpha_{i} + \beta_{i}\right)}{\Gamma\left(\alpha_{i}\right)\Gamma\left(\beta_{i}\right)} \left(\frac{x}{D_{PM}}\right)^{\alpha_{i}-1} \left(1 - \frac{x}{D_{PM}}\right)^{\beta_{i}-1} I_{\left\{0 \le x \le D_{PM}\right\}}$$
(7)

where $i \in [1, N]$ represents the number of PM actions; the relationship between model parameters $[\alpha_i > 0, \beta_i > 0]$ and $[u, \sigma^2]$ is as follows:

$$E\left[\frac{X\left(R_{i}^{+}\right)}{D_{PM}}\right] = \frac{\alpha_{i}}{\alpha_{i} + \beta_{i}} = 1 - \exp\left(-iu\right)$$
(8)

$$Var\left[\frac{X(R_i^+)}{D_{PM}}\right] = \frac{\alpha_i \beta_i}{\left(\alpha_i + \beta_i\right)^2 \left(\alpha_i + \beta_i + 1\right)} = \sigma^2$$
(9)

The parameters, α_i and β_i , can be approximately obtained based on the statistical analysis with the historical failure time data after each PM action and, the parameters, u and σ^2 , in the residual damage model can then be easily estimated by using the Maximum Likelihood Estimation (MLE) method. For detailed procedures of the parameter estimation, readers are referred to the literature [17] and the references therein.

Remark 3: It can be easily proved that the mean of the system residual damage in a replacement cycle is showing an increasing trend with the sequence of PM action, as shown in Eq. (4) and (8). That is, the ability of the maintained oil-lubricated system to operate in a healthy condition is weakening, which is in line with reality.

3.3. The maintenance time model

Recall that a more extend PM duration may be required with the system operation dues to the possible dismantling inspection and the replacement of the potential components [27, 31] since a long time is required to maintain a severely aged system than a slighted aged one. Thus, in this paper, it is assumed that the PM durations shows an increasing trend in a replacement cycle.

To be specific, let us denote the duration needed to implement the *i* th PM action as M_i , and is independent with M_j for any $i \neq j$. Similar to the choice in existing literature [10, 13], it is assumed that the PM durations are exponentially distributed, described as:

$$E[M_i] = \gamma_0 D_{PM} \exp(i\gamma_1 D_{PM})$$
(10)

where $\gamma_0 > 0$ and $\gamma_1 \ge 0$ are constants. In reality, γ_0 and γ_1 can be easily estimated by using the real field data of the PM durations to fit the exponential distribution in Eq. (10).

Remark 4: The commonly used PM duration model in existing literature [4,10] is included in the proposed model as its special case when $\gamma_1 = 0$ is assumed. It can be clearly concluded from Eq. (10) that $E[M_i] \ge E[M_j]$ for any $i > j \ge 1$, which is in line with reality.

Remark 5: The rationality of such modeling assumptions described above is verified in the case study. Please note that such assumptions may not be suitable for all applications, and some other distributions assumption like Weibull distribution may be used in other applications.

4. PM policy formulation and optimization

The oil-lubricated system used in critical systems is usually constrained with mission availability. As mentioned earlier, the PM actions would recover the maintained oil-lubricated system to an intermediate state due to the system aging, i.e., cumulative wear of friction pairs, which results in a more extend PM duration and a shorter expectation of the operating time. Thus, with the residual damage model developed in Section 3, the goal of this section is to develop an availability maximum PM policy for the oil-lubricated system with the average short-run availability constraint.

4.1. The optimization problem formulation

For the oil-lubricated systems monitored using regular oil analysis, the achieved availability can be maximized by controlling the PM threshold. After each system replacement, the oil-lubricated system will be recovered to a good-as-new state. Therefore, the considered optimal PM problem is calculated by the following programming formulation:

MAX Availability_L(
$$D_{PM}$$
) = $\frac{\sum_{i=1}^{N+1} T_i}{\sum_{i=1}^{N+1} T_i + \sum_{i=1}^{N} M_i + \zeta}$
(11)

Subject Teo
$$\begin{cases} 0 < D_{PM} \le D_F \\ Availability _S(i) \ge Availability _S_{min}; i \in [1, N] \\ E\left[\sum_{i=1}^{N+1} T_i\right] \ge T_{total} \end{cases}$$

The second constraint ensures that the average short-run availability is not lower than a threshold. If this constraint elapsed, the oil-lubricated system will be replaced by a new one. The concerned average shortrun availability is expressed as Availability $S(i) = T_{i+1} / (T_{i+1} + M_i)$. The third constraint indicates that the total running time of the oillubricated system should not be less than a predetermined period of

time, T_{total} , as the obtained optimal PM policy is meaningless when the total running time is shorter than a threshold.

4.2. The optimization algorithm description

An optimization algorithm is designed to solve the programming formulation to obtain the optimum PM threshold. Specifically, the optimal PM threshold is calculated by using a searching algorithm to search over the range $(0, D_F]$ that maximizes the achieved Availability _L. The optimization algorithm is as follows.

Step 1: Initialize the PM threshold D_{PM} within the range $(0, D_F]$ with a small value;

Step 2: Calculate the expected operating time, $E[T_{i+1}]$, using Eu. (10), and the expected PM duration, $E[M_i]$, using Eq. (6);

Step 3: Calculate the average short-run availability,

Availability_S(i) = $T_{i+1} / (T_{i+1} + M_i)$, for every PM action until the constraint, Availability_S_{min}, is firstly violated, and set the obtained N as the number of PM actions;

N as the number of PM actions; *Step 4*: Calculated the total running time, $\sum_{i=1}^{N+1} T_i$; if $\sum_{i=1}^{N+1} T_i \ge T_{lotal}$ is satisfied, calculated the achieved availability Availability_L for the current PM threshold D_{PM} , otherwise, go to next;

Step 5: Adjust the PM threshold D_{PM} by a small increment unless the PM threshold $D_{PM} > D_F$ is satisfied and repeat steps 2-4.

Step 6: Choose the optimal D_{PM} with the maximization of the achieved availability Availability_L.

5. Case study

In this section, a practical case study for PSST systems used in large engineering machinery is provided to illustrate the detailed application procedures of the proposed PM policy optimization method and to investigate the effectiveness of the proposed method. The PSST system usually works in severe work conditions with constraints on mission availability, especially for military usage like armored tank vehicles. Recent research shows that about 50% of in-service failures of the PSST systems result from metal wear debris [34,35]. As such, the concentrations of metal wear debris are used to reveal the degradation of the PSST systems, and the oil spectral analysis is used to reveal the metal wear debris in the lubricating oil [14]. For more than 10 years, we have collected the oil field data of the PSST system for more than one thousand samples, of which each dataset contains the time data concerning the system operating, the PM and system replacement, as well as the spectral oil data that represents the degradation evolution of the PSST system. A detailed description of the sampling and analysis processes can be found in [31]. In this way, the optimal PM threshold with required availability constraints can be obtained by using the proposed maintenance policy optimization method.

5.1. Development of the degradation model

Using the above mentioned spectral oil data, the degradation profile of the PSST system can be established and, the expectation of the operating time can then be predicted with the PM threshold using Eq. (6). Since the collected spectral oil data shows increasing but not necessarily a monotonic trend; thus, in this paper, a Wiener processbased degradation model is adopted. To be specific, let X(t) represents the system degradation condition at time t. The degradation is expressed by:

$$X(t) = \theta t + \tau B(t), \quad \theta \ge 0$$
(12)

where B(t) is a standard Brownian movement, $\tau B(t) \sim N(0, \tau^2 t)$, $\tau > 0$ represents the diffusion coefficient; $\theta \ge 0$ is the drift coefficient that characterizes the degradation rate of the monitored oil-lubricated system.

According to the CBM theory, the system residual life is defined based on the first hitting time (FHT) of the Wiener process-based degradation model $\{X(t), t \ge 0\}$. Given the system failure threshold D_f , such FHT can be defined by $T_f = \inf \{t | X(t) \ge D_f\}$ [15]. T_f has an inverse Gauss distribution and the corresponding PDF and cumulative distribution function (CDF) is:

$$f_{T_f}(t) = \frac{D_f}{\sqrt{2\pi t^3 \tau^2}} exp\left(-\frac{\left(D_f - \theta t\right)^2}{2\tau^2 t}\right)$$
(13)

$$F_{T_f}(t) = \Phi\left(\frac{-D_f + \theta t}{\tau \sqrt{t}}\right) + exp\left(\frac{2\theta D_f}{\tau^2}\right) \Phi\left(\frac{-D_f + \theta t}{\tau \sqrt{t}}\right)$$
(14)

Based on the homogeneous Markov property and the independent increment property of the Wiener process, the residual life T_i after the *i* th PM with residual damage $X(R_i^+) < D_f$, can be formulated as:

$$T_{i} = \inf\left\{ \mathsf{t} \mid X(\mathsf{t}) + X(R_{i}^{+}) \ge D_{f} \right\} \text{ if } X(R_{i}^{+}) < D_{f}; \text{ otherwise } T_{X(R_{i}^{+})} = 0$$

Similar to the FHT distribution in Eq. (13) and (14), the residual life T_i knowing $X(R_i^+)$ also conforms to an inverse Gauss distribution. Thus, with the residual damage of the system estimated, the condition PDF and CDF of the residual life, T_i , after the *i* th PM actions can be easily obtained by replacing D_f by $D_f - X(R_i^+)$ in Eq. (13) and (14) as:

$$\begin{cases} f_{T_i}\left(t|X\left(R_i^+\right)\right) = \frac{D_{PM} - X\left(R_i^+\right)}{\sqrt{2\pi t^3 \tau^2}} \exp\left(-\frac{\left(D_{PM} - X\left(R_i^+\right) - \theta t\right)^2}{2\tau^2 t}\right) \\ F_{T_i}\left(t \mid X\left(R_i^+\right)\right) = \Phi\left(\frac{-\left(D_f - X\left(R_i^+\right)\right) + \theta t}{\tau \sqrt{t}}\right) + \exp\left(\frac{2\theta\left(D_f - X\left(R_i^+\right)\right)}{\tau^2}\right) \Phi\left(\frac{-\left(D_f - X\left(R_i^+\right)\right) + \theta t}{\tau \sqrt{t}}\right) \end{cases}$$
(15) (16)

For the PSST system we used here, the wear debris of Cu shows the most contribution to the system failure, according to our previous research in [14]. Therefore, in this paper, we focus on wear debris of Cu (as shown in Fig. 1) to illustrate the proposed method. When the spectral oil data of Cu reaches the failure threshold, $D_F = 0.04\%$, the system is considered failed. Using the spectral oil data with the abovementioned degradation modeling method, the degradation profile of the monitored PSST system can be established, and the system residual life can then be estimated. Specifically, the parameters $\left[\theta, \tau^2\right]$ of the degradation model were estimated using the MLE method. A detailed description of the MLE method can be found in [17]. As such, the degradation model parameters are $\theta = 3.15 \times 10^{-5}$, $\tau^2 = 9.45 \times 10^{-6}$, which will be used to initialize the optimization problem.

Remark 6: Please note that not all the mental wear debris shows the same degradation pattern. Some other types of wear debris may be used in other machines, such as the wear debris of Fe for marine diesel engines [30]. The selection choice of wear debris can be made based on correlation analysis, principal component analysis (PCA), and clustering analysis, as mentioned in [27].

5.2. Estimation of the imperfect PM model

Recall that the PM actions (e.g., dismantling inspection, lubricating oil replacement) do not restore the oil-lubricated system to a goodas-new state but an intermediate state due to the system aging property such as cumulative wear. As a result, the ability of the PM actions to keep the system operates in a healthy condition is weakening. In other words, the system aging property will shorten the expected operating time and prolong the PM duration. Thus, in order to establish a realistic optimization model for the considered PSST system, the expected operating time and the PM durations of the considered PSST system are estimated according to the description in Section 4.

5.2.1. The operating time model

The operating time model is estimated using the operating time data collected during the mission. A preliminary analysis shows that the time period in service between the PM actions shows a decreasing trend, i.e., $E[T_i] > E[T_j]$ for i < j. Such orders following the sequences of PM actions describe the relative degree of the system aging by the current PM action to the previous ones, and lead to the decisions on PM implementation and, eventually, the system replacement. Thus, the historical failure time data following the sequences of PM actions are used to estimate the parameters $[u, \sigma^2]$ in the system residual damage model according to the proposed method in Section 3.1. As such, the model parameters are u = 0.672, $\sigma^2 = 0.008$, which represents the effort and the variance of the PM actions.

Remark 7: Please note that the determination method for the system residual damage is an open issue in the existing literature, especially for the complex systems with multiple components, which is not the research focus of this paper. Of course, other distribution such as Weibull distribution can also be used according to the system aging property of the monitored system. For detailed methods of system residual damage estimation, readers are referred to the literature [18] and the references therein.

5.2.2. The maintenance time model

The PM duration model is estimated using the maintenance duration data collected during the mission. A preliminary analysis including Anderson-Darling test [21] shows that the PM durations in a replacement cycle shows an exponentially increasing trend, i.e., $E[M_i] \ge E[M_j]$ for i > j, which is in line with our proposed maintenance time model. Therefore, we estimated the parameters using the collected field data of the PM durations to fit the exponential distribution in Eq. (9). As such, the model parameters are $\gamma_0 = 1.54$, $\gamma_1 = 0.357$.

Remark 8: It is noted that a more extend PM duration is required with the system operating. That because a longer time is required to maintain an aged system than a slighted aged one due to the possible dismantling inspection and potential components replacement. Of course, other models, such as a linear decreasing model, can also be used according to the PM durations of the monitored system.

5.3. Solution of the optimal PM policy

With the estimated expectations of the operation time and PM durations, the established PM policy optimization model can be initialized, and the optimal PM threshold can be eventually obtained. For the sake of illustration, the parameters of the

Parameters	Values	Parameters	Values	
D_f	0.04‰	и	0.672	
Availability_S _{min}	0.6	σ^2	0.08	
T _{total}	60(Day)	ζ	3.5(Day)	
θ	3.15×10 ⁻⁵	γ ₀	1.54	
τ^2	9.45×10 ⁻⁶	γ1	0.357	

optimization model are shown in Tab. 1.

To be specific, the proposed PM optimization problem can be understood in the following way: The degradation of the monitored PSST system is monitored using regular oil spectral analysis and modeled using a Wiener process with the parameters $\theta = 3.15 \times 10^{-5}$, $\tau^2 = 9.45 \times 10^{-6}$. When the collected spectral oil data reaches the threshold $D_F = 0.04\%$, the system is defined as failed. The system is replaced based on system failure or the short-run availability *Availavility*_{S_{min} = 0.6 cannot be sustained, and a time $\zeta = 3.5$ (Day) is required. On the other hand, the system is preventively maintained upon a threshold D_{PM} with lubricating oil replacement and possible dismantling inspection and components replacement [35]. The effects of the PM actions are characterized by u = 0.672, $\sigma^2 = 0.008$, (see}



Fig. 5. System residual damage after PM actions

Eq. (4) and (5)). And the PM durations are characterized by $\gamma_0 = 1.54$ and $\gamma_1 = 0.357$, according to Eq. (10).

According to Eq. (4) and (5), the system residual damage, $X(R_i^+), i = 1, 2, ...$, is shown in Fig. 5 with the number of PM actions increase. It can be seen that the estimated system residual damage shows an increase with the rise of the number of PM actions. When the number of the PM actions reaches 5, the system residual damage will rise to the PM threshold, D_{PM} . It is noted that frequent PM actions will be performed if the system residual damage approaches the PM threshold, D_{PM} , too much. This phenomenon will lead to a violation of the short-run availability constraint. So, the PM threshold should be optimized to satisfy the mission requirement.

Since the threshold, D_{PM} , is a decision variable in engineering practice as well as our proposed method; the objective is to determine the optimal PM threshold. Above all, the problem is formulated as: MAX Availability L(D PM)

Subject to
$$\begin{cases} 0 < D_{PM} \le 0.04\%\\ Availability _S(i) \ge 0.6; \quad i \in [1, N]\\ E\left[\sum_{i=1}^{N+1} T_i\right] \ge 60 \end{cases}$$

After calculation, the average short-run availability after multiple PM actions with various D_{PM} as well as the number of PM actions is

Number of PM actions	$D_{PM} (\%)$									
	0.004	0.008	0.012	0.016	0.02	0.024	0.028	0.032	.0.036	0.04
0	Total service time $< T_{cont}$				1	1	1	1	1	1
1					0.973	0.973	0.973	0.973	0.973	0.973
2					0.947	0.943	0.938	0.932	0.925	0.918
3					0.875	0.868	0.856	0.842	0.829	0.817
4	- Total			0.766	0.750	0.722	0.687	0.646	0.589	
5				0.658	0.632	0.576	0.525	0.448	0.397	
Total service time (Day)					65.7	70.2	74.7	79.4	85.5	92.4

Table 2. Short-run availability $Availability_S$ with different PM threshold for the PSST system

presented in Tab. 2. In addition, the total service time is presented for different PM threshold, D_{PM} , with an increment of 0.004‰. The cases when the PM threshold, D_{PM} , less than 0.02‰ are excluded for that the corresponding total service time is violated with the required time constraint, $T_{total} \ge 60$ (Day).

In Tab. 2, the bold font indicates the smallest number of PM actions, N, when the short-run availability constraint is violated. The evolution of the achieved availability, Availability_L, with the increase of PM threshold, D_{PM} , is shown in Fig. 6. The optimal maintenance policy with the maximized long-run availability is finally obtained as:

 $D_{PM}^* = 0.024\%, N^* = 5$, and the associated long-run availability, Availability $L^* = 0.876$.



Fig. 6. The achieved availability with PM threshold

Table 3. Sensitive analysis of the model parameters on the optimal threshold

to consider the accurate record of the information concerning system operation and maintenance.

6. Conclusions and discussions

In this paper, a maintenance policy optimization method is proposed for oil-lubricated systems based on the selected oil analysis data. Most of the existing maintenance policies focused on minimizing the maintenance cost without considering the mission constraints and the system aging property. Compared with these existing methods, the proposed method considers the system aging property, as well as the mission constraints such as the short-run availability, such that the achieved system long-run availability of the maintained system can be maximized and therefore, constitute the novelty of this paper. The proposed method was verified for several PSST systems, and the results show that the proposed method is practical, effective and robust.

The obtained results are of practical significance for determining the optimal PM threshold and, thus, consists of the main contribution of this paper. The system aging property and the corresponding operation time models make the maintenance model more practical and more comfortable to implement, which is another contribution of this paper. In addition, the possible applications of the proposed method are much wider. For example, it can be used for other oil-lubricated systems with mission constraints, such as the wind turbine gearbox in wind power systems [23], and the diesel engine in marine freighters [11]. Moreover, the obtained outcomes can also complement the researches of system residual life prediction; for example, the derived results in the works of Liu et al. [25], Vališ et al. [19], Yan et al. [31]. Following the method proposed in this paper, these previous results might be complemented when used in lubricant condition monitoring, wear failure evaluation and other Prognostics and Health Management (PHM) applications.

The main contribution of this paper is not only a new direction in the maintenance policy optimization for oil-lubricated systems but also open up possibilities for evaluating the system residual damage using other historical operation data. There are several possible directions deserving future research. First, the

 σ^2 и Yo Y1 Variation -10% 10% -10% 10% -10% 10% -10% 10% -10% 10% Optimal threshold 0.024 0.024 0.028 0.024 0.028 0.024 0.024 0.028 0.024 0.024 $D_{PM}^{*}(\%)$

Furthermore, it can be seen from Fig. 6 that, with the PM threshold, D_{PM} , increasing, the achieved availability may increase even though the short-run availability is decreasing (e.g., at the PM threshold $D_{PM} = 0.02\%$ and 0.024% in Fig. 6). This phenomenon may result from the balance of the system uptime expectation and downtime expectation. Compare with the PM threshold, $D_{PM}^{*'} = 0.02\%$ that determined empirically [35], the optimal PM threshold in our paper can obtain a higher achieved availability and a longer service time.

Since the parameters, u, σ^2 , ζ , γ_0 and γ_1 , describing the optimization model are estimated based on the population-wide characteristics from historical operation data, the variations of such parameters for a particular system may affect the optimization results. Thus, in order to analyze the influence of the parameters to the optimal result, a sensitivity analysis of the model parameters of the PM durations and replacement time to the PM policy is investigated, as shown in Tab. 3. It can be seen that the optimal PM threshold is slightly sensitive to the parameter variations of the PM duration model (e.g., γ_0 and γ_1), and the operating time model (e.g., u), and is insensitive to the system replacement time (e.g., ζ). As such, it can be concluded that the system aging property and the PM durations description have important influences on the planning of the maintenance policy. Therefore, we need mechanism-based system aging property description method might be developed to complement the historical failure data-based method proposed in this paper. Second, other distributions may have to be considered to establish the operating time model and the malignance time model in other cases. Third, more degradation modeling methods that can fuse multiple condition monitoring data may have to be used when modeling other systems. Fourth, other maintenance modeling method that can deal with the cases with storage conditions should be investigated in future research.

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References

- Abbou A, Makis V. Group maintenance: A restless bandits approach. INFORMS Journal on Computing 2019; 31(4): 719-731, https://doi. org/10.1287/ijoc.2018.0863.
- Ahn H S, Yoon E S, Sohn D G, Kwon O K, Shin K S, Nam C H. Practical contaminant analysis of lubricating oil in a steam turbine-generator. Tribology international 1996; 29(2): 161-168, https://doi.org/10.1016/0301-679X(95)00003-M.
- 3. Barde S R, Yacout S, Shin H. Optimal preventive maintenance policy based on reinforcement learning of a fleet of military trucks. Journal of Intelligent Manufacturing 2019; 30(1): 147-161, https://doi.org/10.1007/s10845-016-1237-7.
- 4. Crowder M, Lawless J. On a scheme for predictive maintenance. European Journal of Operational Research 2007; 176(3): 1713-1722, https://doi.org/10.1016/j.ejor.2005.10.051.
- 5. Du Y, Wu T, Makis V. Parameter estimation and remaining useful life prediction of lubricating oil with HMM. Wear 2017; 376: 1227-1233, https://doi.org/10.1016/j.wear.2016.11.047.
- Du Y, Wu T, Zhou S, Makis V. Remaining useful life prediction of lubricating oil with dynamic principal component analysis and proportional hazards model. Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology 2019; 1350650119874560, https://doi.org/10.1177/1350650119874560.
- Fan C Y, Fan P S, Chang P C. A system dynamics modeling approach for a military weapon maintenance supply system. International Journal of Production Economics 2010; 128(2): 457-469, https://doi.org/10.1016/j.ijpe.2010.07.015.
- Gajewski J, Vališ D. The determination of combustion engine condition and reliability using oil analysis by MLP and RBF neural networks. Tribology International 2017; 115: 557-572, https://doi.org/10.1016/j.triboint.2017.06.032.
- 9. Guinat E. Spectrometric oil analysis: atomic emission spectrometric analysis of wear metals in lube oils by RDE method. Tribology international 1985; 18(4): 246, https://doi.org/10.1016/0301-679X(85)90070-2.
- Guo C, Wang W, Guo B, Si X. A maintenance optimization model for mission-oriented systems based on Wiener degradation. Reliability Engineering & System Safety 2013; 111: 183-194, https://doi.org/10.1016/j.ress.2012.10.015.
- Jun H B, Kiritsis D, Gambera M, Xirouchakis P. Predictive algorithm to determine the suitable time to change automotive engine oil. Computers & Industrial Engineering 2006; 51(4): 671-683, https://doi.org/10.1016/j.cie.2006.06.017.
- Li X, Makis V, Zuo H, Cai J. Optimal Bayesian control policy for gear shaft fault detection using hidden semi-Markov model. Computers & Industrial Engineering 2018; 119: 21-35, https://doi.org/10.1016/j.cie.2018.03.026.
- Liao H, Elsayed E A, Chan L Y. Maintenance of continuously monitored degrading systems. European Journal of Operational Research 2006; 175(2): 821-835, https://doi.org/10.1016/j.ejor.2005.05.017.
- Liu Y, Ma B, Zheng C S, Li S C. Research and evaluation on wear in power-shift steering transmission through oil spectral analysis with RKPCA method. Spectroscopy and Spectral Analysis 2015; 35(5): 1370-1375.
- 15. Liu Y, Ma B, Zheng C S, Xie S Y. Failure prediction of power-shift steering transmission based on oil spectral analysis with Wiener process. Spectroscopy and Spectral Analysis 2015; 35(9): 2620-2624.
- 16. Makis V, Wu J, Gao Y. An application of DPCA to oil data for CBM modeling. European Journal of Operational Research 2006; 174(1): 112-123, https://doi.org/10.1016/j.ejor.2005.03.010.
- 17. Myung I J. Tutorial on maximum likelihood estimation. Journal of Mathematical Psychology 2003: 47(1): 90-100, https://doi.org/10.1016/ S0022-2496(02)00028-7.
- Osman T, Velex P. Static and dynamic simulations of mild abrasive wear in wide-faced solid spur and helical gears. Mechanism and Machine Theory 2010; 45(6): 911-924, https://doi.org/10.1016/j.mechmachtheory.2010.01.003.
- Prabhakaran A, Jagga C R. Condition monitoring of steam turbine-generator through contamination analysis of used lubricating oil. Tribology International 1999; 32(3): 145-152, https://doi.org/10.1016/S0301-679X(99)00028-6.
- 20. Raposo H, Farinha J T, Fonseca I, Galar D. Predicting condition based on oil analysis-A case study. Tribology International 2019; 135: 65-74, https://doi.org/10.1016/j.triboint.2019.01.041.
- 21. Scholz F W, Stephens M A. K-sample Anderson-Darling tests. Journal of the American Statistical Association 1987; 82(399): 918-924, https://doi.org/10.1080/01621459.1987.10478517.
- 22. Sheng S. Monitoring of wind turbine gearbox condition through oil and wear debris analysis: A full-scale testing perspective. Tribology Transactions 2016); 59(1): 149-162, https://doi.org/10.1080/10402004.2015.1055621.
- Tian Z, Jin T, Wu B, Ding F. Condition based maintenance optimization for wind power generation systems under continuous monitoring. Renewable Energy 2011; 36(5): 1502-1509, https://doi.org/10.1016/j.renene.2010.10.028.
- Vališ D, Gajewski J, Žák L. Potential for using the ANN-FIS meta-model approach to assess levels of particulate contamination in oil used in mechanical systems. Tribology International 2019; 135: 324-334, https://doi.org/10.1016/j.triboint.2019.03.012.
- Vališ D, Žák L. Contribution to prediction of soft and hard failure occurrence in combustion engine using oil tribo data. Engineering Failure Analysis 2017; 82: 583-598, https://doi.org/10.1016/j.engfailanal.2017.04.018.
- Vališ D, Žák L, Pokora O. Perspective approach in using anti-oxidation and anti-wear particles from oil to estimate residual technical life of a system. Tribology International 2018; 118: 46-59, https://doi.org/10.1016/j.triboint.2017.09.017.
- Vališ D, Žák L, Pokora O, Lánský P. Perspective analysis outcomes of selected tribodiagnostic data used as input for condition based maintenance. Reliability Engineering & System Safety 2016; 145: 231-242, https://doi.org/10.1016/j.ress.2015.07.026.
- Wakiru J M, Pintelon L, Muchiri P N, Chemweno P K. A review on lubricant condition monitoring information analysis for maintenance decision support. Mechanical Systems and Signal Processing 2019; 118: 108-132, https://doi.org/10.1016/j.ymssp.2018.08.039.
- 29. Wang W. A prognosis model for wear prediction based on oil-based monitoring. Journal of the Operational Research Society 2007; 58(7): 887-893, https://doi.org/10.1057/palgrave.jors.2602185.
- 30. Wang W, Hussin B, Jefferis T. A case study of condition based maintenance modelling based upon the oil analysis data of marine diesel engines using stochastic filtering. International Journal of Production Economics 2012; 136(1): 84-92, https://doi.org/10.1016/j.ijpe.2011.09.016.
- Yan S, Ma B, Zheng C. Health index extracting methodology for degradation modelling and prognosis of mechanical transmissions. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2019; 21(1): 137-144, https://doi.org/10.17531/ein.2019.1.15.

- 32. Yan S. F., Ma B., Zheng C. S. Condition-based maintenance for power-shift steering transmission based on oil spectral analysis. Spectroscopy and Spectral Analysis 2019; 39(11): 3470-3474.
- Yan S, Ma B, Zheng C, Chen J. An optimal lubrication oil replacement method based on selected oil field data. IEEE Access 2019; 7: 92110-92118, https://doi.org/10.1109/ACCESS.2019.2927426.
- 34. Yan S F, Ma B, Zheng C S, Zhu L A, Chen J W, Li H. Z. Remaining useful life prediction of power-shift steering transmission based on uncertain oil spectral data. Spectroscopy and Spectral Analysis 2019; 39(2): 553-558.
- 35. Zheng C, Liu P, Liu Y, Zhang Z. Oil-based maintenance interval optimization for power-shift steering transmission. Advances in Mechanical Engineering 2018; 10(2): 1687814018760921, https://doi.org/10.1177/1687814018760921.
- 36. Zhu J, Yoon J M, He D, Bechhoefer E. Online particle-contaminated lubrication oil condition monitoring and remaining useful life prediction for wind turbines. Wind Energy 2015; 18(6): 1131-1149, https://doi.org/10.1002/we.1746.

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